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STUDY OF THE CHARACTERISTICS OF SCATTERING OF ELECTROMAGNETIC WAVES FROM COMPLEX-SHAPED BODIES BY THE METHOD OF AUXILIARY SOURCES

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The problem of diffraction of a plane wave on "sphere–cone–sphere" and "cone–sphere" bodies by the method of auxiliary sources is solved. According to this method, diffraction fields may be presented in the form of vector wave potentials with densities distributed over the auxiliary surface drawn inside the scattering body. Comparison of theoretical and experimental results is made.

Consider the case of incidence of a plane electromagnetic wave $\overline{E}_0(\overline{r})$, $\overline{H}_0(\overline{r})$ onto a closed perfectly conducting surface S with the normal $\overline{n}$ determined unambiguously. To solve the problem of diffraction, the method of auxiliary sources is used [1], according to which the scattered fields $\overline{E}_1(\overline{r})$, $\overline{H}_1(\overline{r})$ are represented by vector wave potentials with densities in the form of electric $\overline{I}_{\mathrm{el}}(\overline{r})$ and (or) magnetic $\overline{I}_{\mathrm{mag}}(\overline{r})$ currents distributed over a certain auxiliary surface S_0 that lies entirely inside the body at which diffraction occurs:

$$\overline{E}_1(\overline{r}) = \frac{\xi}{ik}\,\mathrm{rotrot}\iint_{S_0} \overline{I}_{\mathrm{el}}(\overline{r}')\, g(k\,|\overline{r}-\overline{r}'|)\, ds' + \mathrm{rot}\iint_{S_0} \overline{I}_{\mathrm{mag}}(\overline{r}')\, g(k\,|\overline{r}-\overline{r}'|)\, ds' ,$$

$$\overline{H}_1(\overline{r}) = \mathrm{rot}\iint_{S_0} \overline{I}_{\mathrm{el}}(\overline{r}')\, g(k\,|\overline{r}-\overline{r}'|)\, ds' - \frac{1}{ik\xi}\,\mathrm{rotrot}\iint_{S_0} \overline{I}_{\mathrm{mag}}(\overline{r}')\, g(k\,|\overline{r}-\overline{r}'|)\, ds' ,$$

where $g(k\,|\,\overline{r}-\overline{r}'|) = \dfrac{\exp(ik\,|\overline{r}-\overline{r}'|)}{4\pi\,|\overline{r}-\overline{r}'\,|}$ is the Green's function of the free space and $\xi = \sqrt{\mu_0/\varepsilon_0}$ is the wave resistance of the free space.

Operation with rot is carried out on the variables without primes (i.e., not on the integration variables). Scattered fields can be represented only with the aid of electric magnetic currents in the case of k^2 where is not an eigenvalue of the corresponding internal boundary-value problem for the region bounded by the surface S_0 [1]. Moreover, the auxiliary surface must enclose singular points that appear when the diffraction field is extended analytically toward the inside of the body. For convenience of the numerical solution, unlike [2], the scattered fields will be described with the aid of magnetic currents $\overline{I}_{\mathrm{mag}}$. As is seen from the expressions given above for the secondary fields, when the scattered electric fields are calculated with the use of electric currents, it is necessary to operate with rot twice, whereas with the use of magnetic currents it should be done once. Therefore, the representation of scattered fields by means of magnetic currents allows one to simplify the algorithm and reduce the number of operations in the calculations. It should be noted that the auxiliary magnetic currents $\overline{I}_{\mathrm{mag}}$, just as the auxiliary electric currents $\overline{I}_{\mathrm{el}}$, do not have a physical meaning, but the field generated by them coincides exactly with the true diffraction field outside the scatterer by virtue of the uniqueness theorem. As a boundary condition we use the relation $[\,\overline{n}, \overline{E}_1\,] = [\,\overline{n}, \overline{E}_0]$, assuming that

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the conductivity of the surface S is perfect and, consequently, the tangential component of the total electric field vanishes there. Replacing $\bar{E}_1$ by the expression in terms of $\bar{I}_{\text{mag}}$, we obtain the following equation for determining the surface magnetic currents:

$$\left[\bar{n}\,(\bar{r})\,,\,\text{rot}\iint\limits_{S_0}\bar{I}_{\text{mag}}\,(\bar{r}')\,g\,(k\,|\bar{r}-\bar{r}'|)\,ds'\right]=\left[\bar{n}\,(\bar{r})\,,\,\bar{E}_0\,(\bar{r})\right],\quad r\in S\,. \tag{1}$$

To determine the scattering characteristics, it is convenient to use the vector amplitude of scattering $\bar{F}$, which depends on the direction of incidence $\bar{k}_{\text{inc}}$ and the direction of observation $\bar{k}_{\text{obs}}$ and which is determined with the aid of the relation

$$\bar{E}_1\,(\bar{r})=\bar{F}\,(\bar{k}_{\text{obs}}\,,\,\bar{k}_{\text{inc}})\,\frac{\exp\,(-\,ikr)}{r}\,,\quad kr>>1\,.$$

In this case, according to the optical scheme, the total section for scattering is determined from the expression

$$\sigma^{\text{tot}}=\frac{4\pi}{k}\,\text{Im}\,[\bar{e}_0\,,\,\bar{F}\,(\bar{k}_{\text{obs}}\,,\,\bar{k}_{\text{inc}})]\,, \tag{2}$$

in which $\bar{e}_0$ is the unit vector that characterizes the direction of polarization of the incident wave.

The radiolocation section for scattering is determined by the relation

$$\sigma^{\text{rad}}=4\pi\,|\bar{e}_0\,,\,\bar{F}\,(-\,\bar{k}_{\text{inc}}\,,\,\bar{k}_{\text{inc}})|^2\,. \tag{3}$$

If we decompose the reflected signal into two orthogonal linearly polarized modes, then expression (3) will describe the power contained in that mode whose polarization coincides with the polarization of the original wave. The power of the transverse polarized signal is obtained from relation (3) by substituting $\bar{e}'_0$ = $[\bar{k}_{\text{inc}},\ e_0]$ for $\bar{e}_0$.

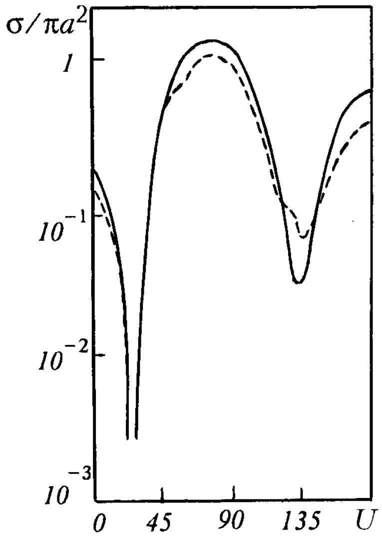
On the basis of expressions (1)–(3), a computational algorithm was developed for solving the problem of diffraction of electromagnetic waves on bodies of conical shape. The coordinate origin is located at the center of the sphere circumscribed around the scatterer. To obtain a numerical solution, the auxiliary surface S_0 was subdivided into N cells.

The problem of finding the surface currents $\bar{I}_{\text{mag}}$ is reduced to the solution of the system of linear equations by imposing the requirement of equality of the left- and right-hand sides of Eq. (1) at N points of the scatterer and replacing the integral by a sum following the rule of rectangles. Out of $3N$ scalar equations appearing in this case for calculating $3N$ tangential components of magnetic currents, only $2N$ are linearly independent owing to the fact that div $\bar{E}$ = 0. In constructing the dependence of the radiolocation section for scattering from the observation angle, multiple solution of linear systems with the same matrix and different right-hand sides is required; therefore, to obtain $\bar{I}_{\text{mag}}$, the method of matrix reversal by the Kraut algorithm was used and also the conversions of the matrix in the L–V form [3].

As a first example, consider a body of the type "sphere–cone–sphere" with the following parameters: semivertex angle 15$^{\text{o}}$, ratio of the radii of the front and rear (respectively, smaller and larger) spheres

$$b/a=1/(1+\sin 15^{\circ})\approx 0.7944\quad\text{and}\quad ka=1.5\,,\quad k=2\pi/\lambda\,.$$

In the calculations, auxiliary surfaces of two types were used: one similar to the scattering one with a certain similarity factor α and an equidistant surface offset from the scattering one by a distance βa. The results obtained from calculations with these auxiliary surfaces, whose parameters are related as $\beta = 1 - \alpha$, turned out to be close at a not very large β. This is due to the compactness of the body, i.e., the ratio of its



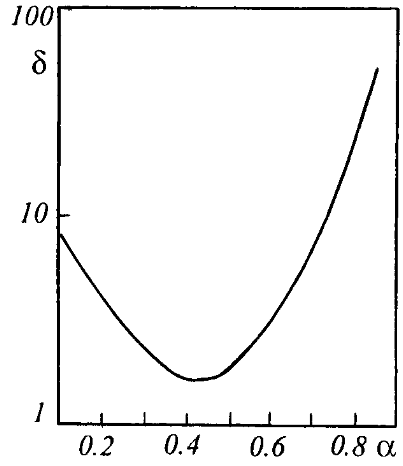


Fig. 1. Dependence of the radiolocation section for scattering of the sphere–cone–sphere, normalized to the area of the diametral section of a large sphere, on the angle of incidence (degrees). The solid curve represents theory; the dashed curve is the experiment.

Fig. 2. Dependence of the uncertainty in the satisfaction of boundary conditions (δ, % of the incident wave amplitude) on the coefficient of similarity α.

length to its width is equal to 1.3 and two surfaces at $\beta = 1 - \alpha$ are closely spaced. Therefore, due to the simplicity of realization, the trends were investigated with the use of an auxiliary surface, which is similar to the scattering one. The surface was subdivided into cells in the following way: its conical part was divided into belts by planes, which are perpendicular to the symmetry axis and the distance between which was chosen so that the areas of the belts were the same; each of the spherical parts was divided into spherical belts so that the polar angle in the coordinate system reckoned from the sphere center could obtain the same increment in passing from layer to layer. The belts obtained were divided by azimuth into a specified number of parts, with the parameters of division being selected so that the areas of the cells were approximately identical. The requirement for the method to be stable [1] imposes a restriction on the selection of the auxiliary surface: in order that the singular points (the centers of the spheres) be inside the sphere, the latter must enclose the segment $[-a/2,\ b - a/2]$. To satisfy this requirement, the similarity coefficient α must exceed 0.39.

The results of calculations for the body suggested for consideration are presented in Fig. 1, which presents the diagram of the radiolocation section for scattering as a function of the incidence angle for the case of parallel polarization of the incident wave. The experimental data of [4] were used for comparison. The curves have a maximum near 75° from the direction of specular reflection from the cone part of the surface. It is seen from the figure that there is a good agreement between the calculated and measured values, especially if we take into consideration that the agreement holds in the dynamic range of 20 dB.

In the case considered, the optimal value of α was determined to obtain reliable results with a minimum N, i.e., with the smallest expenditures of computer time. For this purpose, computation was made for $\alpha \in [0.1;\ 0.8]$ with a step of 0.1 on grids with 52, 112, and 152 meshes.

The reliability of the results obtained was evaluated by controlling the accuracy with which the boundary conditions and the reciprocity theorem were fulfilled. The minimum errors (at a fixed number of meshes N) correspond to the most accurate solution, which is obtained for an optimum position of the auxil-

TABLE 1. Dependence of the Uncertainty in Determining the Coefficient of Similarity on the Number of Meshes

Number of meshes N	Coefficient of similarity α				
	0.4	0.5	0.6	0.7	0.8
112	7.3	3.3	0.14	1.6	3.1
152	0.9	0.9	0.64	0.16	2.9

iary surface. The accuracy with which the boundary conditions (of the residual) were fulfilled was controlled at the nodes, which are intermediate to those at which they were set. The dependence of the uncertainty with which the boundary conditions are satisfied on the similarity coefficient α (i.e., on the position of the auxiliary surface) for the case of 152 meshes is given in Fig. 2. The similarity coefficients α are laid on the abscissa axis and the maximum values of the residual that were obtained in calculation of diagrams are laid on the ordinate axis. It is seen from the figure that the value of the error is minimum at α = 0.4–0.5, i.e., when the auxiliary surface is located near the singular points. For N = 52, the position of the minimum practically did not change, whereas the value of the residual increased, as was expected. At N = 112, the position of the minimum was displaced by about 0.1 to the left.

In the calculations, the error in the fulfillment of the reciprocity theorem was controlled; according to this theorem, the amplitude of the wave scattered forward must not change on reversal of the direction of incidence. By virtue of the axial symmetry of the scatterer, this corresponds to the reflection of the direction of incidence relative to the plane normal to the symmetry axis. To control the error, total sections for scattering were computed and the uncertainty was defined as the ratio of their difference to the half-sum in the case of axial incidences (0 and 180^o), since in this case maximum divergences were fixed. Table 1 provides understanding of the fulfillment of the reciprocity theorem.

It is seen from the table that on increase of N the auxiliary surface, which corresponds to the minimum of uncertainty, shifts to the scattering surface, which may be due to the singularity contained in the Green's function of the free space.

Since in the case where N = 152 the results of computations at α = 0.6 and 0.7 turned out to be nearly the same, we may assume, on the basis of Table 1, that the optimum coefficient of similarity lies near 0.6. To check this assumption, computations were performed with N = 52. The comparative analysis of the data obtained at α = 0.5 and α = 0.7 showed that the accuracy of computations turned out to be satisfactory, while at α = 0.6 the relative error did not exceed 11% when N = 52 had been replaced by N = 152; the value at the central maximum was 1.4%. The result obtained is in complete agreement with the conclusion made in [5] for a two-dimensional case, in conformity with which, at not very high requirements placed on the accuracy of the computations, it is worthwile to place an auxiliary surface approximately in between the singular points and the scatterer surface.

The computations made showed that the method gives good results in a certain region of the values of α which expands with increase in the number of meshes. At N = 52, this is the small region near α = 0.6, and for N = 152 this is the segment [0.2; 0.7].

As a second example, we consider a sharp cone with a spherical base. Formally, one cannot apply here the method of auxiliary sources, because the normal to the surface is discontinuous on the cone vertex. However, it is known that if a scattering surface is deformed so that the size of deformation is much smaller than the incident wavelength, then the scattering characteristics will remain as they were before, i.e., the wave does not note the change in the scatterer surface. This fact allows tapering of the cone. The radius of this tapering must be substantially smaller than the characteristic dimensions of the body and of the incident wavelength. From this it becomes clear that the auxiliary surface must not be shifted far from the scattering one, at least near the vertex, as it must enclose the centers of the spheres for the method to be stable. This

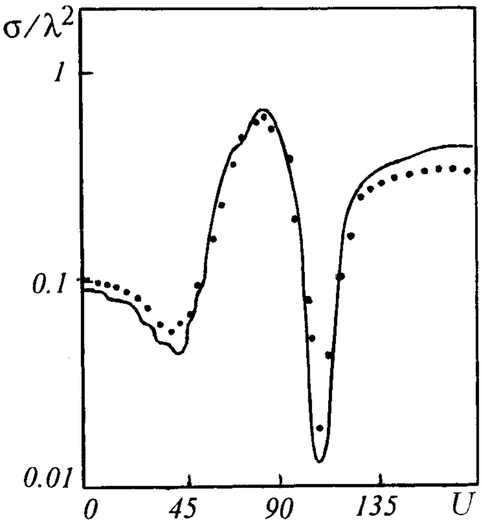


Fig. 3. Dependence of the radiolocation section for scattering of the cone–sphere, normalized to the square of the length of the incident wave (degrees).

and the requirement of preventing poor conditionality of the matrix allow the conclusion that here it is convenient to use an equidistant auxiliary surface.

The surface was divided into zones along the symmetry axis just as before: the conical surface was divided into belts of equal area and the spherical ones uniformly over the angle. The meshes were obtained by uniform division of the zones by the azimuthal angle.

The characteristics of scattering were computed for a cone–sphere with the following parameters: the semivertex angle of the cone is 15^{o}, $ka = 1.7$ (a is the radius of the sphere). Investigations of the convergence of the method depending on the radius of curvature were carried out in relation to the body indicated. The best results were obtained for a curvature radius of about $0.2a$. The dependence of the radiolocation section for scattering on the incidence angle for perpendicular polarization of the incident wave is shown for this case in Fig. 3 by dots. The solid line in this figure corresponds to the experimental data of [6].

As a result of the investigations carried out, a computational procedure has been developed for predicting the characteristics of scattering of electromagnetic waves on complex-shaped bodies. The reliability of the procedure is confirmed by the good coincidence of the theoretical and experimental data. The method implemented makes it possible to calculate, with a good degree of accuracy, the radiolocation characteristics of small-size bodies of complex noncoordinate shape. The number of meshes required for obtaining a reliable result depends strongly on the way of selecting an auxiliary surface. In view of this, the time of numerical solution, approximately proportional to N^3, can change by more than an order of magnitude.

NOTATION

$\overline{E}_0(\overline{r})$ and $\overline{H}_0(\overline{r})$, incident electrical and magnetic fields; $\overline{E}_1(\overline{r})$ and $\overline{H}_1(\overline{r})$, scattered electrical and magnetic fields; $\overline{r}$, radius-vector; r', integration variable; r, coordinate on the surface S; $\overline{n}$, external normal; $\overline{I}_{el}$ and $\overline{I}_{mag}$, electric and magnetic currents; $\overline{k}$, wave vector; k, wave number; λ, wavelength of the field; $\overline{F}$, vector amplitude of scattering; $\overline{e}_0$ and $\overline{e}'_0$, unit vectors that characterize polarization of electromagnetic waves; μ_0 and ε_0, magnetic and dielectric permeabilities of free space; σ^{tot}, total section for scattering; σ^{rad}, radiolocation section for scattering; U, angle of incidence of an electromagnetic field; δ, error; N, number of meshes on the auxiliary surface.

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